Relevance-driven Evaluation of Modular Nonmonotonic Logic Programs

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Knowledge-Based Systems Group

Modularity in Logic Programming

- Splitting set [Lifschitz and Turner, 1994]
- Generalized Quantifiers [Eiter *et al.*, 1997]
- Templates [Ianni *et al.*, 2003]
- Macros [Baral *et al.*, 2006]
- Smodels program modules [Oikarinen and Janhunen, 2008], DLP-functions [Janhunen *et al.*, 2007]
- Incremental modularity [Gebser *et al.*, 2008]
- Modular Nonmonotonic Logic Programs
  [D_, Eiter, Fink, Krennwallner, 2009]

Modular Nonmonotonic Logic Programs

- Analogies to ordinary programming languages
  - Function prototype
  - Formal parameters
  - Function body
  - Function calls
  - Call by reference/value

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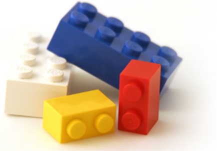
  - Mutually recursive calls between modules
  - Semantics based on instantiations
- Naive evaluation is infeasible in practice

Evaluation of MLPs



Modular Nonmonotonic Logic Programs

Stratification & Splitting for MLPs

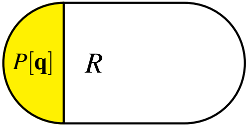
Algorithm

Syntax of MLPs

Module atoms $P[p_1, \ldots, p_k].o(t_1, \ldots, t_\ell)$

Rules: $\alpha_1 \vee \cdots \vee \alpha_k \leftarrow \beta_1, \ldots, \beta_j, \text{not } \beta_{j+1}, \ldots, \text{not } \beta_m$

Modules $m = (P[\mathbf{q}], R)$



Modular Nonmonotonic Logic Programs $\mathbf{P} = (m_1, \ldots, m_n)$

Example: Odd/Even

$\mathbf{P} = (m_1, m_2, m_3)$, where $m_1 = (P_1, R_1)$, $m_2 = (P_2[q_2], R_2)$, $m_3 = (P_3[q_3], R_3)$.

$$R_1 = \{q(a). \quad q(b). \quad ok \leftarrow P_2[q].even.\}$$

$$R_2 = \left\{ \begin{array}{lcl} q_2'(X) \vee q_2'(Y) & \leftarrow & q_2(X), q_2(Y), \\ & & X \neq Y. \\ skip_2 & \leftarrow & q_2(X), \text{not } q_2'(X). \\ even & \leftarrow & \text{not } skip_2. \\ even & \leftarrow & skip_2, P_3[q_2'].odd. \end{array} \right\}$$

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$main()$:
$n := |q|$
if $even(n)$ **then return** ok

$even(n)$:
$n' := n - 1$
if $n' < 0$ **then return** true
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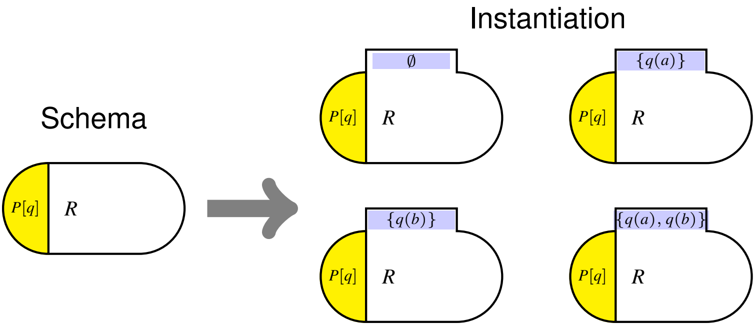
M
$M_1/\emptyset : \{ok, q(a), q(b)\}$
$M_2/\{q_2(a), q_2(b)\} : \left\{ \begin{array}{l} even, skip_2, \\ q_2(a), q_2(b), q_2'(b) \end{array} \right\}$
$\vdots$
$M_2/\emptyset : \{even\}$
$\vdots$
$M_3/\{q_3(b)\} : \{odd, skip_3, q_3(b)\}$
$\vdots$

Instantiations of modules



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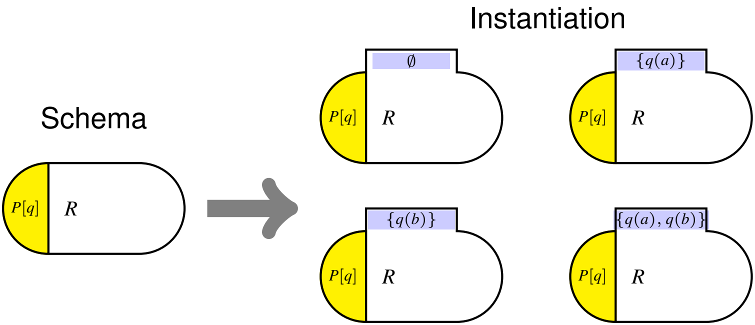
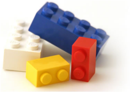


Value call: $P[S]$, with input $S \subseteq HB_{\mathbf{P}}|_{\mathbf{q}}$

$VC(\mathbf{P})$: set of all value calls $P[S]$ in $\mathbf{P}$



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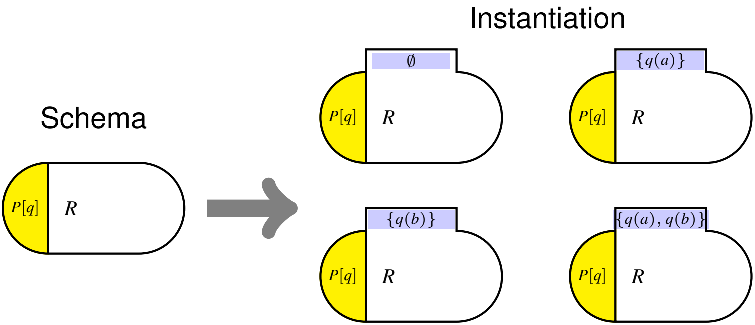

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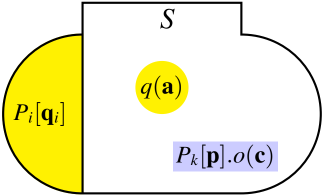
$I_{\mathbf{P}}(P[S]) = R \cup S$ is an instantiation of $m = (P[\mathbf{q}], R)$ with $S \subseteq HB_{\mathbf{P}}|_{\mathbf{q}}$

The rule base $I(\mathbf{P}) = (I_{\mathbf{P}}(P_i[S]) \mid P_i[S] \in VC(\mathbf{P}))$ is the instantiation of $\mathbf{P}$

Interpretation and Models



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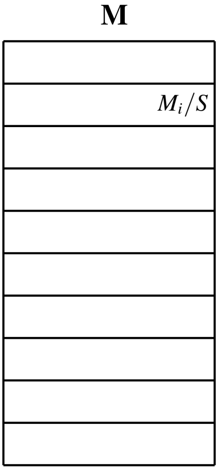

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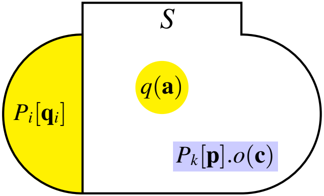

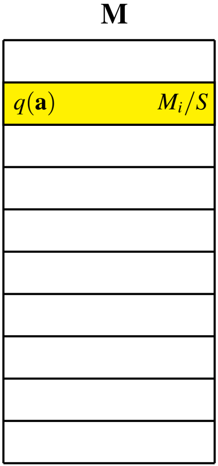

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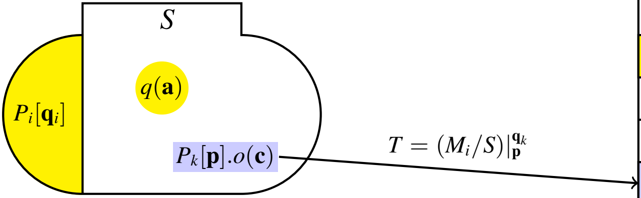

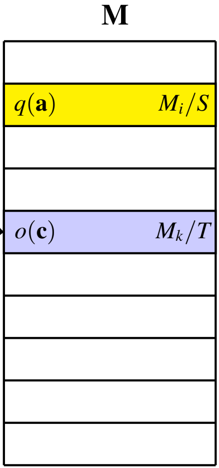

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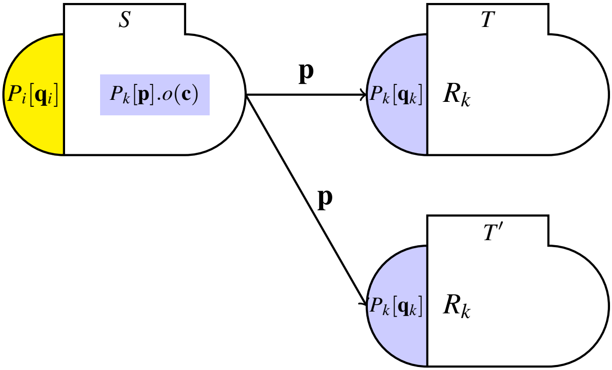
$\mathbf{M}, P_i[S] \models P_k[\mathbf{p}].o(\mathbf{c})$ iff $o(\mathbf{c}) \in M_k/T$

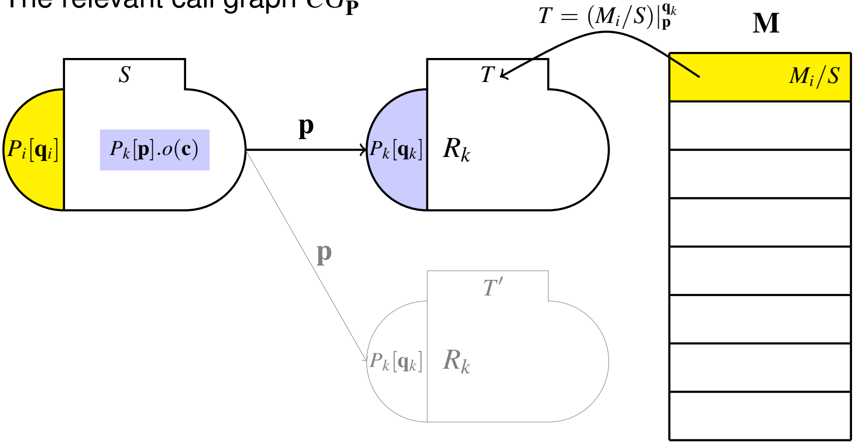
Call graphs and Relevant call graphs

The call graph $CG_{\mathbf{P}}$

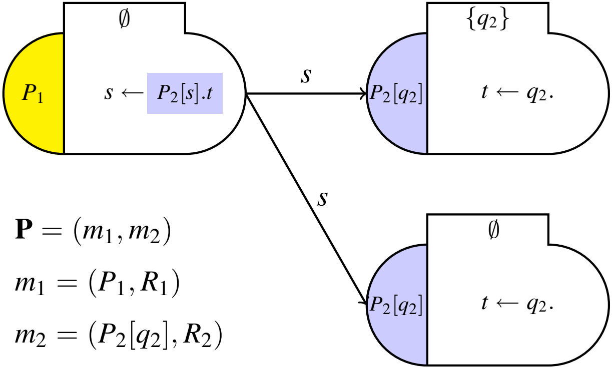


Call graphs and Relevant call graphs

The relevant call graph $CG_{\mathbf{P}}^{\mathbf{M}}$



Example



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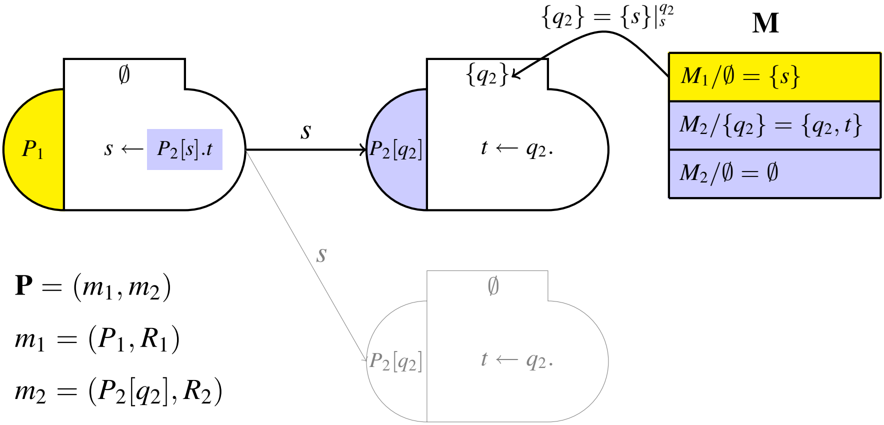

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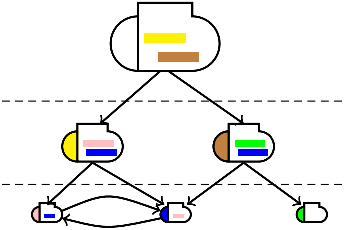
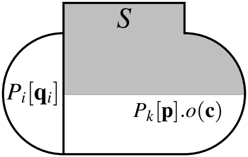
$\mathbf{M}$ is an answer set of $\mathbf{P}$ wrt C for $\mathbf{M}$ iff $\mathbf{M}$ is a minimal model of $f\mathbf{P}^{\mathbf{M},C}$

MLP stratifications at two different levels



The global level along the relevant call graph



The local level (inside module instances) based on the (instance) dependency graph

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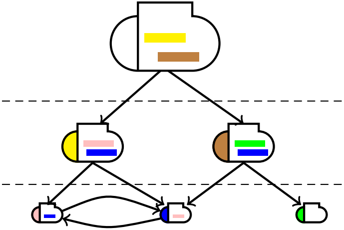

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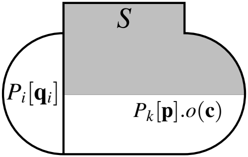

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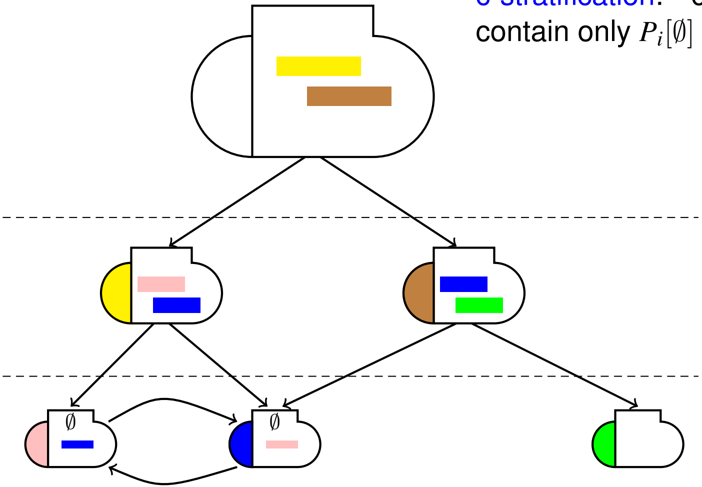
c-stratification: cycles in $CG_{\mathbf{P}}^{\mathbf{M}}$ contain only $P_i[\emptyset]$ nodes



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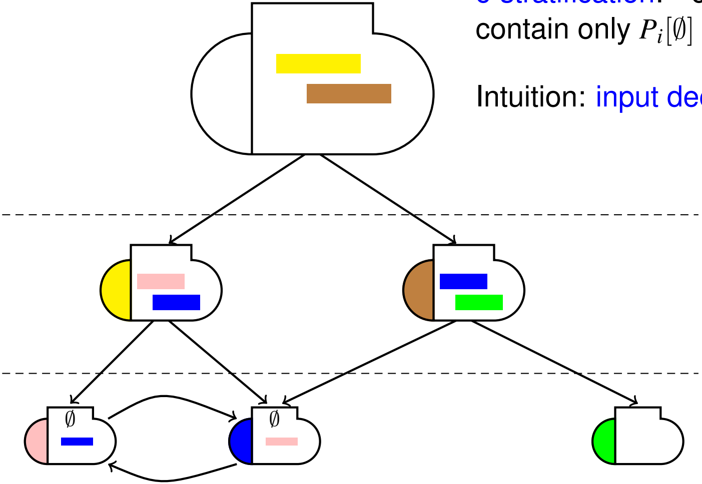

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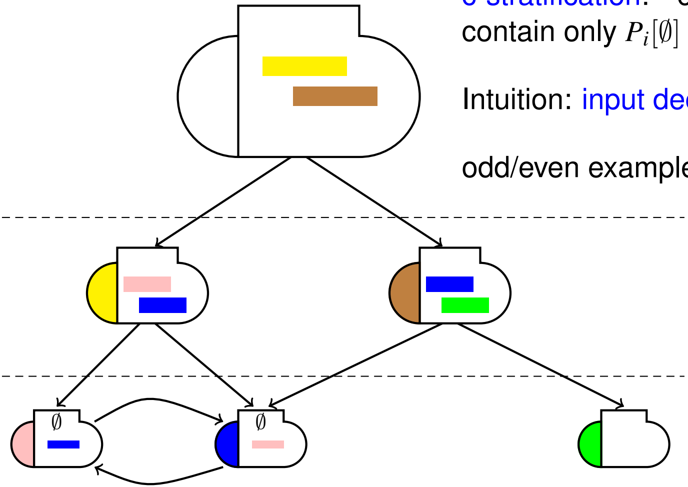

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Intuition: input decreasing

odd/even example: c-stratified

input-stratification: (locally) split inside modules

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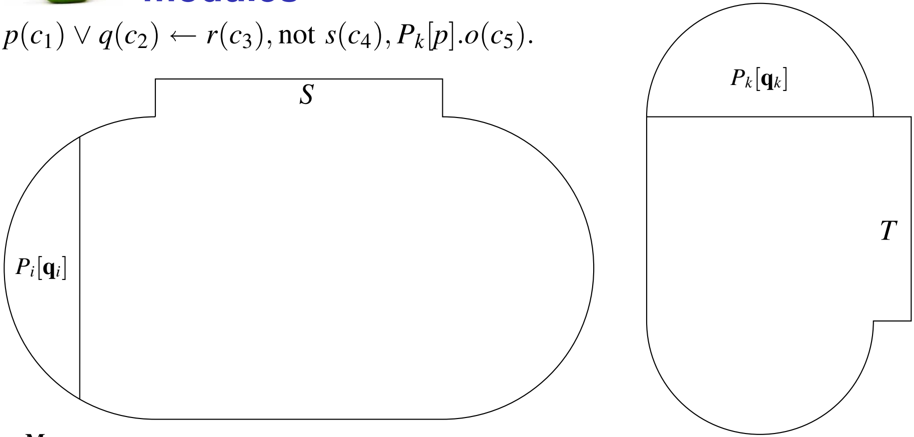

$G_{\mathbf{P}}^{\mathbf{M}}$: the instance dependency graph

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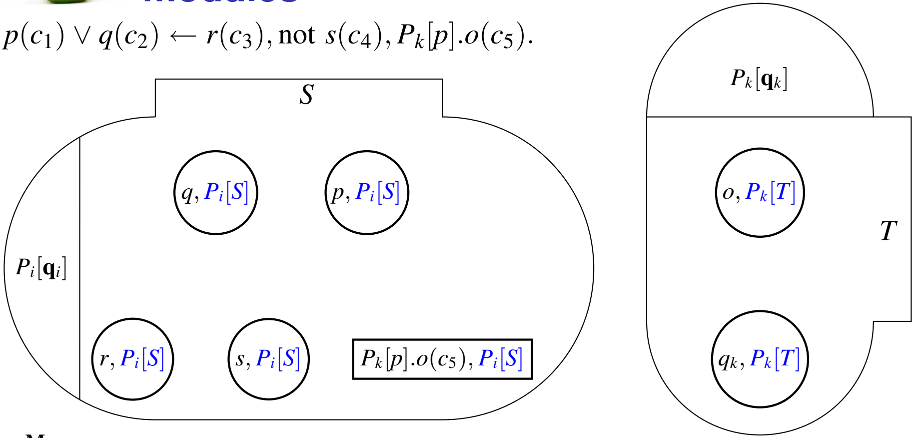

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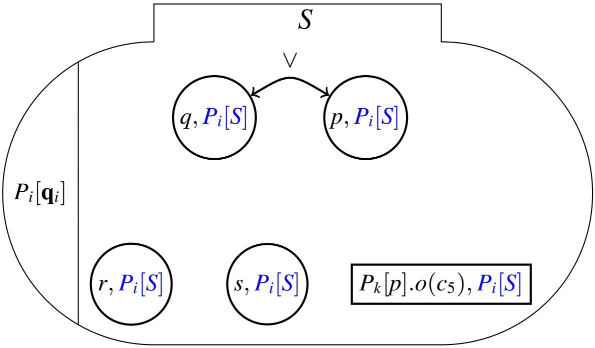

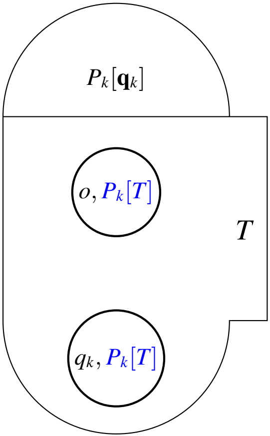

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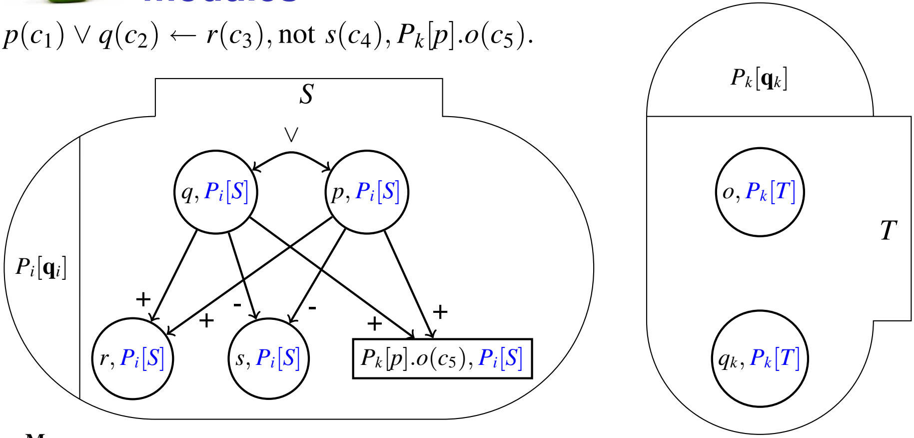

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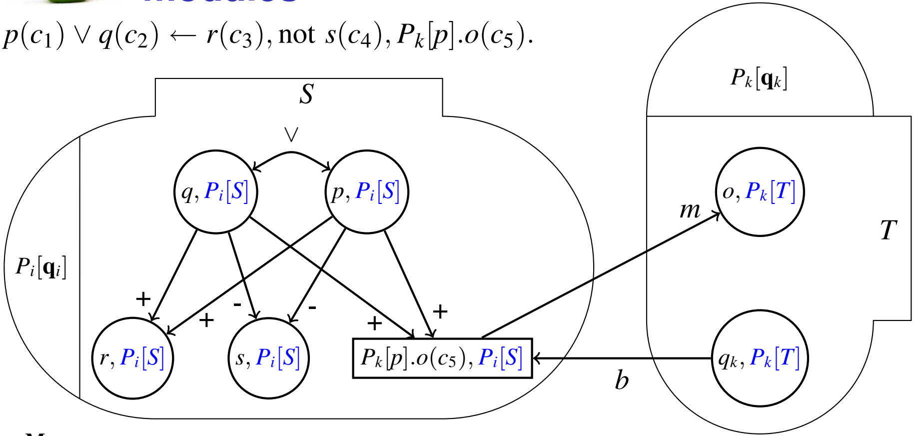

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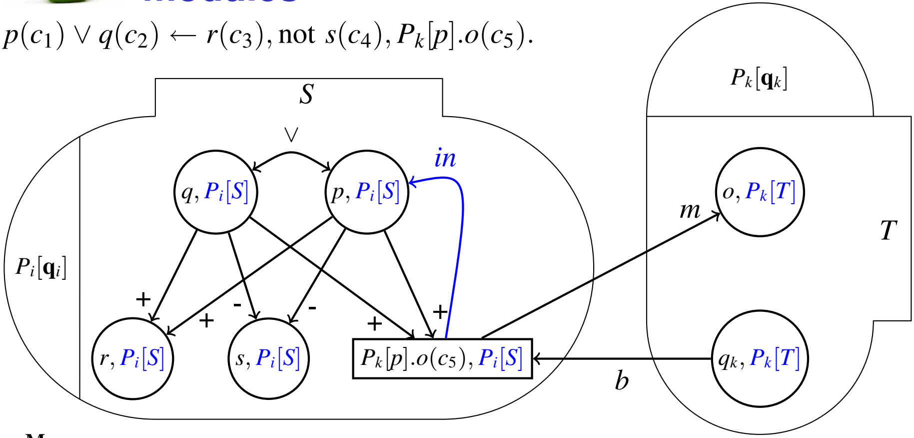

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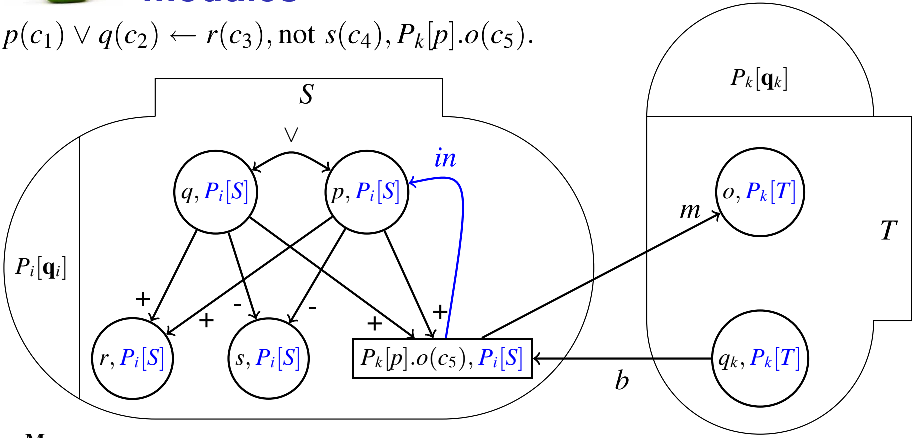

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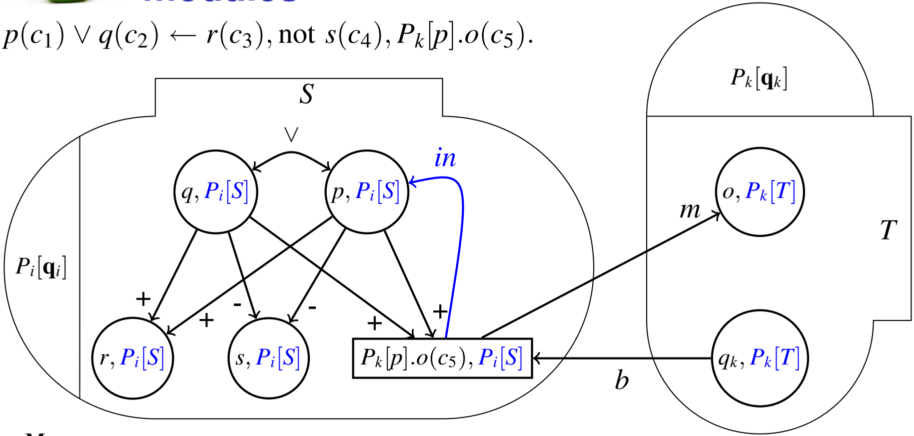

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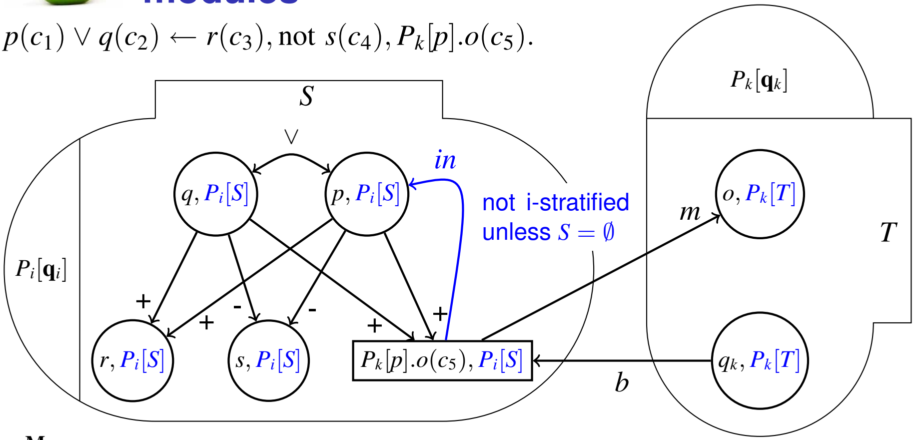

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Extending splitting sets to MLPs

- Given
  - $\mathbf{P}$: an MLP
  - R: set of ground rules
  - $\mathbf{p} = p_1, \ldots, p_\ell$: list of predicate names
- $def(\mathbf{p}, R) = \{p_i(\mathbf{d}) \mid \exists r \in R, p_i(\mathbf{d}) \in H(r), p_i \in \mathbf{p}\}$

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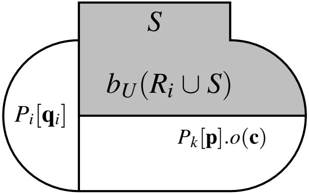

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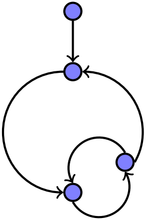
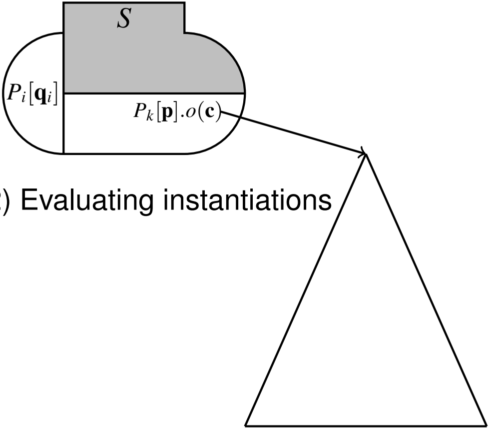
Top-down evaluation algorithm

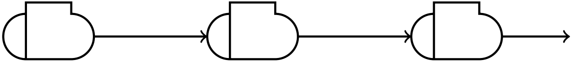


(1) Detecting cycles



(2) Evaluating instantiations

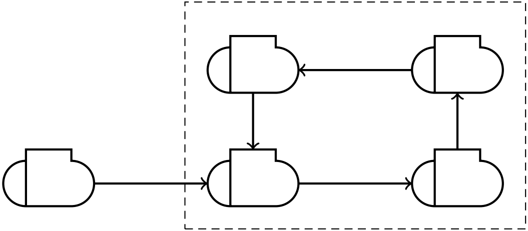
Detecting cycles



Done by maintaining a “path” of sets of visited value calls.

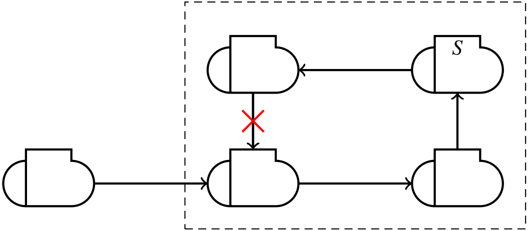
Each element of “path” is initialized with a single value call.

Detecting cycles



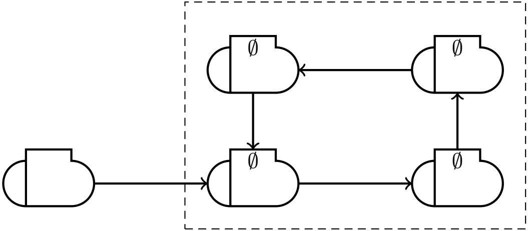
When a cycle is detected, there are two cases:

Detecting cycles



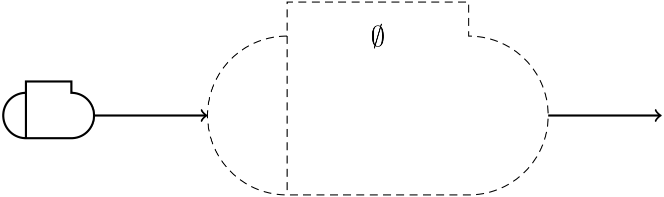
(1) A member of the cycle has non-empty input, we backtrack.

Detecting cycles



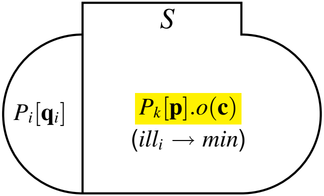
(2) All members of the cycle have empty input

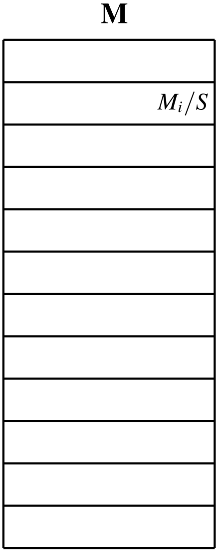
Detecting cycles



We combine all value calls of the cycle and treat the result as a new "virtual" value call.

Evaluating module instances





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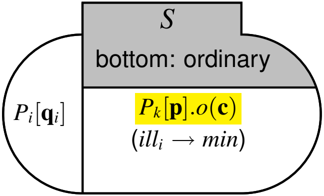

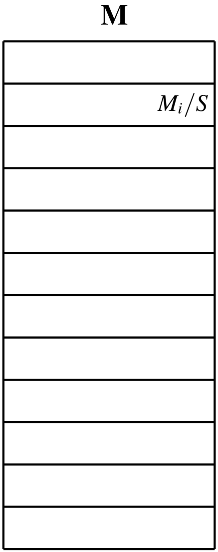

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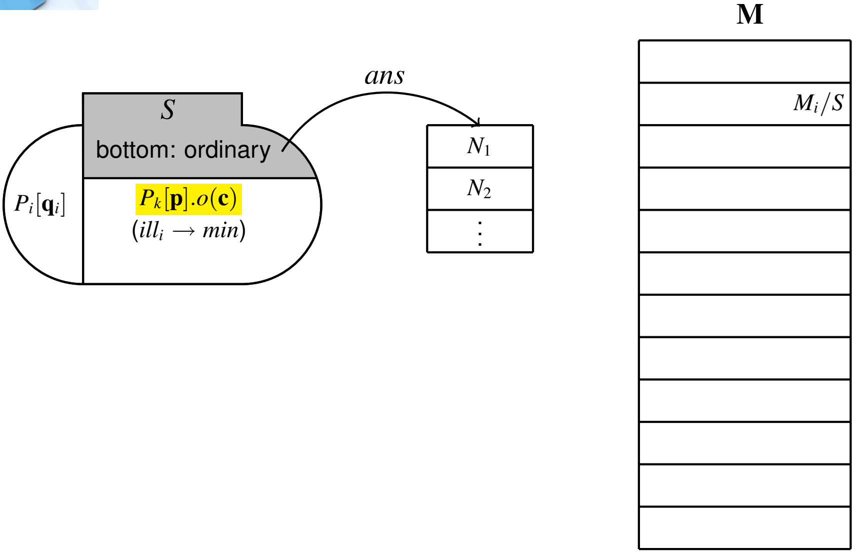

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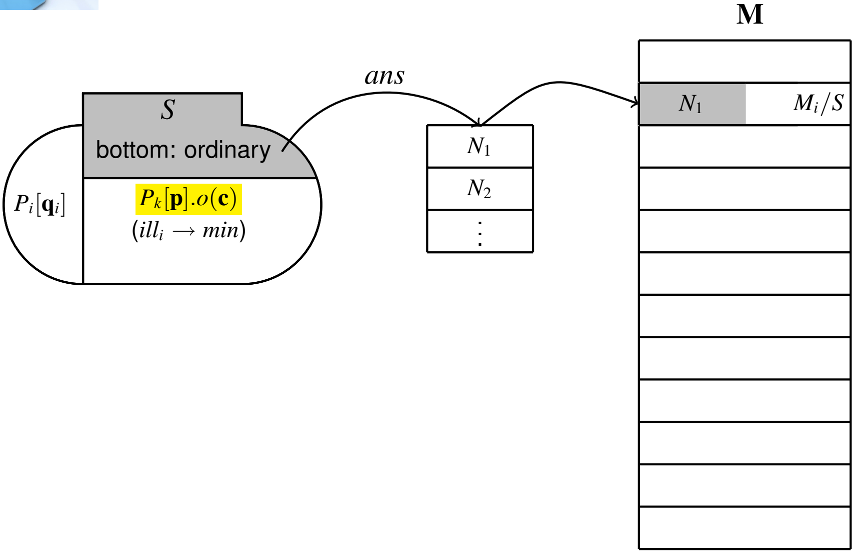

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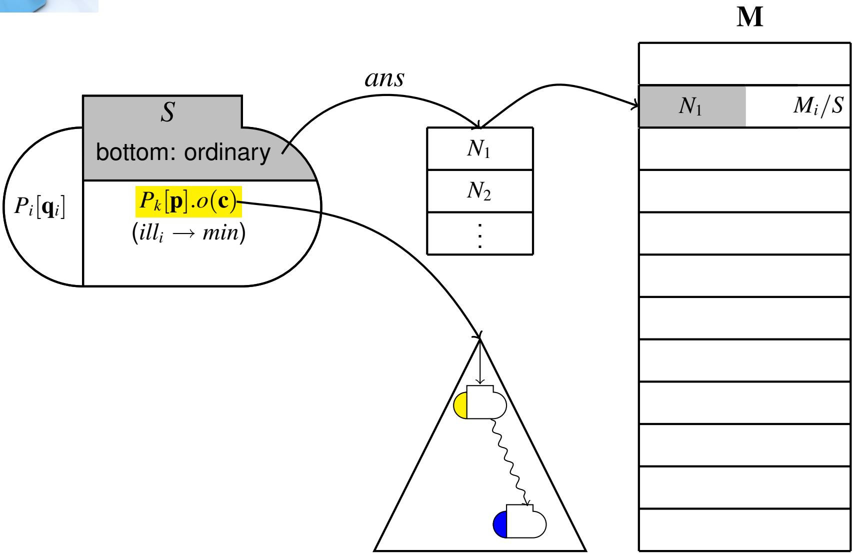

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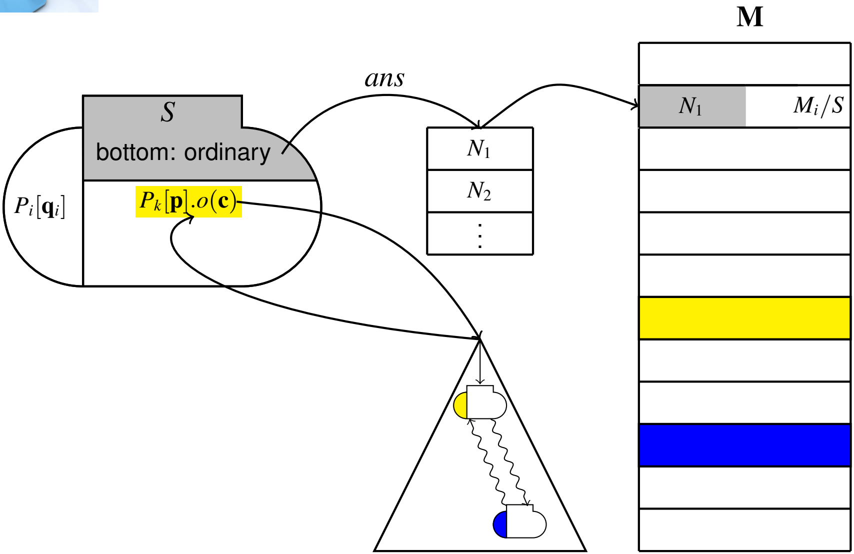

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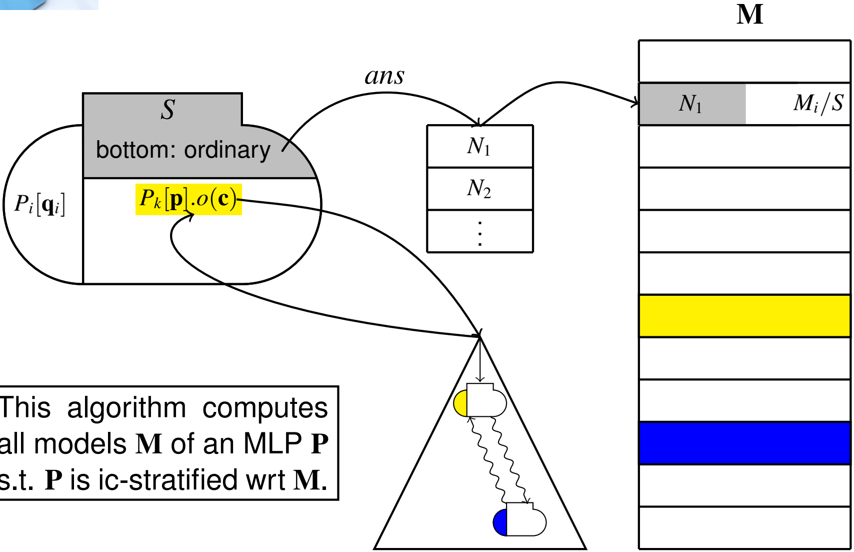

This algorithm computes all models $\mathbf{M}$ of an MLP $\mathbf{P}$ s.t. $\mathbf{P}$ is ic-stratified wrt $\mathbf{M}$.

Conclusion

We investigated

- Different notions of stratification for MLPs
  - call-stratification (global - along the relevant call graph)
  - input-stratification (local - inside modul instances)
- Generalize the Splitting theorem for ic-stratified MLPs
- A top-down algorithm for evaluating ic-stratified MLPs

Details in the paper:

- i-stratification at the schematic level
- safety condition

Future work

- Extension on stratification
- Implementation

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